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ABSTRACT
        This paper sets out a model that generates new,
relevant information about the relationship among educational output,
scale, and perpupil costs. According to the author, this information,
if used, would result in more efficient educational spending.
Specifically, a long run average cost function is estimated. Minimum
perpupil operating costs are achieved when average daily attendance
is 1,426 students and the total square feet of the facility is
124,062. Such a model has particular relevance for decisions
pertaining to investment in new facilities and/or consolidation.
(Author)

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THE ESTIMATION OF A LONG RUN

COST FUNCTION FOR WEST

VIRGINIA PUBLIC HIGH SCHOOLS

A Paper Presented At

The 1972 Annual Meeting

of the

American Educational Research Association

by

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One of the most important contributions of
economics to educational administration is
a general concept of costs. Such a concept
has important implications for a conceptual
approach to decision-making. It is also a
valuable tool for the practicing administrator.[1]

Introduction: The Need To Consider
Alternative Criteria In The Alloca-
tion Of Educational Resources

Public schools today face increasing financial pressure and what appears to be a growing fiscal crisis resulting from certain endogenous and exogenous changes. The most significant endogenous change has been the relatively rapid rise of powerful teacher organizations which have increased school operating costs considerably in recent years through successful negotiations of higher teacher salaries and certain fringe benefits. Exogenously, there exists presently a public preoccupation with various notions of accountability, and an ongoing change in preference away from public education to other goods and services as evidenced by the increasing propensity towards non-support of public education.[2] These changes have placed increasing pressure on public education, in particular educational administrators, to continue providing qualitatively similar educational programs with relatively fewer resources and higher input prices.

This growing fiscal crisis has several ramifications, one of which is particularly relevant here. Traditional criteria used by educational administrators in making expenditures and allocating resources may, out of immediate necessity and/or long run considerations, need to be reordered, added to, or possibly thrown out entirely and altogether new criteria established (if we wish to maintain or improve the existing quality of educational services presently being provided). More specifically, increasing system needs relative to insufficient resources suggests changed administrative behavior which emphasizes, for a given quality of educational output, expenditure criteria which reflect, to a greater extent than is presently the case, what I call 'enlightened efficiency factors'.[3]

If new criteria are to be established or existing criteria reformulated
(resulting in greater efficiency in educational spending which presently
appears appropriate), educational administrators must begin to identify and
estimate new and relevant information pertinent to this need. This paper
formulates and estimates a long run cost function for selected public high
schools in West Virginia. The cost function generates information potenti-
ally useful in decisions regarding new investment in high school facilities
and/or consolidation of existing ones. The information, if used, would
result in more efficient educational spending and a certain reduction in
system wide costs; thus providing educational administrators alternative
additional criteria information relevant to investment and consolidation
decisions. Specifically, the model indicates for the population being
sampled: (1) the relationship between per-pupil operating cost and educa-
tional output; (2) the optimum level of educational output (if one exists);
(3) the optimum size or scale of operations; (4) the scale of operations for
all output levels which minimizes per pupil costs; and (5) the extent of
economies of scale.

<div align="center">
Theoretical Framework: The Short Run,
The Long Run, The Relationship Between
Cost And Output, The Long Run Cost
Function And Economies Of Scale
</div>

The Short Run And The Long Run

Theoretically, the analysis of cost is couched in one of two 'period
dimensions' -- either the short run or the long run. That is, the analysis
and conclusions regarding a firm's[4] decisions and economic behavior will
differ significantly depending upon whether the firm is assumed to be opera-
ting in the short run or the long run. Understanding of the long run and

the short run and the relationship between the two becomes extremely import-
ant when one moves from the theoretical discussion of cost, cost functions
and economies of scale to the empirical estimation of cost functions from
real world data; therefore a discussion of both and the relationship between
the two is included here.

Neither the short run nor the long run refer to calendar or clock time.
This is why they are often referred to as 'period dimensions', rather than
time periods. The two period dimensions, instead, refer to the time neces-
sary for individuals and firms to adapt fully to new conditions. For instance,
it cannot be said in advance and without reference to a specific problem or
situation that a one or two month period lies in the short run, or that a
ten year period extends into the long run.

Definitions of the short run and long run are ultimately concerned with
the degree of variability of different productive inputs. For a given period
of production, inputs are classified as either 'fixed' or 'variable'. A
fixed input is usually defined as one whose quantity cannot be easily or
readily changed (either increased or decreased) when conditions indicate a
change is either necessary or desirable. It is realized that no input is
ever fixed in the absolute sense, irregardless of how short the period of
time under consideration; but it is assumed that certain inputs can be
treated as constant or fixed, reasoning that the cost of immediate variation
(either an increase or decrease) of these 'fixed' inputs is so large that
it places them well beyond consideration as a possible solution to the par-
ticular problem being considered. Such things as buildings, major pieces
of capital equipment and highly competent administrative personnel are
examples of inputs that cannot be rapidly increased.[5]

Thus a fixed input, while being necessary for production, does not change

with respect to changes in the quantity of output which is produced. A

'variable' input, on the other hand, is one which can be easily and readily

changed in response to desired or necessary changes in the level of output.

The fundamental difference then between fixed and variable inputs is

essentially temporal; in that inputs which are fixed for one period of time

are variable for a longer period of time. Thus, over a sufficiently long

period of time, all inputs are variable.[6]

Given the above distinction between fixed and variable inputs the short

run is defined as that period of time in which the amount or level of one

or more of the productive inputs is fixed.[7] Two additional statements give

further insight into the theoretical-analytical construct of the short run:

> ..the short run is taken to be a period which is long enough to
> permit any desired change in output technologically possible
> without altering the scale of plant, but which is not long enough
> to permit of any adjustment of scale of plant.[8]

> No variation in plant size will be made to adjust to fluxuations
> in output in the short run...so there is no foregone alternative
> to the more intensive use of plant. The only foregone alternative
> is the amount spent on additional units of variable services (inputs).
> When we say that in the short run some inputs are freely variable,
> we mean that their quantity can be varied without affecting their
> price (for a given quality). When we say that other inputs are
> not freely variable, we mean that their quantities can be varied
> within the given time unit -- be it a month -- a week -- or a year
> -- only at a considerable change in their price....Fixidity and
> variability are a matter of degree. In order to simplify the
> formal theory, economists define 'the short run' as a period within
> which some inputs are variable, others fixed. Clearly there are
> many short runs, and the number of freely variable productive
> services increases as the period of time is lengthened.[9]

Thus, the short run refers to a particular situation where at least one

of the inputs in the productive process is fixed; implying that increases in

output can only be achieved by either increasing the use of variable inputs and/or utilizing the fixed input more intensively. No change in the scale of plant is possible in the short run to achieve desired changes in output.

As an example of the short run, consider the situation where a particular school system is faced with a sizeable and unexpected increase in demand for the educational services it provides. The only way the school system can meet this increased demand (in the short run), if we consider existing buildings as the fixed input and teachers as the variable input, is to utilize both teachers and buildings more intensively and/or hire new teachers. The school system in the short run cannot meet this increased demand by increasing its scale of operations; that is, through construction of new classrooms, gymnasiums or cafeterias.

In contrast, the long run refers to that period of time (or planning horizon) in which all inputs are variable. It refers to that time in the future when output changes can be achieved in the manner most advantageous to the administrator.[10] Elsewhere,[11] the long run is defined as

> a period long enough to permit each producer (administrator) to make such technologically possible changes in the scale of his plant (facilities) as he desires, and thus to vary his output either by a more or less intensive utilization of existing plant, or by varying the scale of his plant or by some combination of these methods.

Leftwich[12] says of the long run:

> The long run presents no definitional difficulties. It is a period of time long enough for the firm to be able to vary the quantities per unit of time of all resources used. Thus, all resources are variable. No problem of classifying resources as fixed or variable exists. The firm has sufficient time to vary its scale of plant as it desires from very small to large or vice versa. Infinitesimal variations in size are usually possible.

Returning to our example of the school system faced with an unexpected increase in client demand; the long run would be a period sufficiently long to allow school administrators to respond to the increased demand for educational services in virtually any manner they deemed appropriate (including increasing the scale of operations). Thus, in the long run zero constraints prevail.

The relationship between the long run and the short run. The relationship between the long run and short run becomes crucial in determining the estimation procedure used to arrive at a long run cost function for the sampled population. It will become evident later that while analytically and theoretically there exists a long run; in the real world, where data must be collected, all production and all economic activity take place in the short run.[13] Thus, if we are (as in this case) attempting to empirically estimate theoretically postulated long run hypotheses and relationships from real world cross-sectional data which is short run, an understanding of the relationship between the two is absolutely essential. It is further evident that if no relationship exists between the long run and short run, empirical studies purporting to explain long run phenomena can have little meaning if based on data reflecting short run economic activity.

Fortunately, there does exist a relationship between the long run and the short run. Viewed slightly differently the long run refers to the fact that administrators can plan ahead and choose any and as many aspects of the short run they deem either necessary or appropriate. Thus, for a given problem or situation, the long run consists of all possible short run situations among which an administrator or economic agent may choose.[14] This relationship allows one to move from the short run to the long run by defining all possible short run situations for a particular problem. In our

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case it allows us to empirically estimate long run phenomena from data which, in the real world, reflects short run activity.

The Relationship Between Cost And Output
In The Short Run

The cost curve or cost function expresses the relationship between cost and output over some specified range of output for a given firm. Seven types of costs are usually defined for the firm in the short run:

1. Total fixed cost = quantity of the fixed productive input times its price.

2. Total variable cost = quantity of the variable productive input times its price.

3. *Total cost = total fixed cost plus total variable cost.

4. Marginal cost = increase in total cost divided by the increase in output.

5. Average fixed cost = total fixed cost divided by output.

6. Average variable cost = total variable cost divided by output.

7. *Average cost = average fixed cost plus average variable cost = total cost divided by output, or total cost (3.) divided by output.

The shapes of these various cost curves in the short run depend upon assumptions one makes about input prices and the production process. Specific short run cost functions estimated from real world data may assume a variety of shapes. One possibility which exhibits the properties[15] usually assumed by economists is given in Figures 1 and 2:[16]

TC*

MC

AC*

AVC

AFC

COST ($)

COST ($)
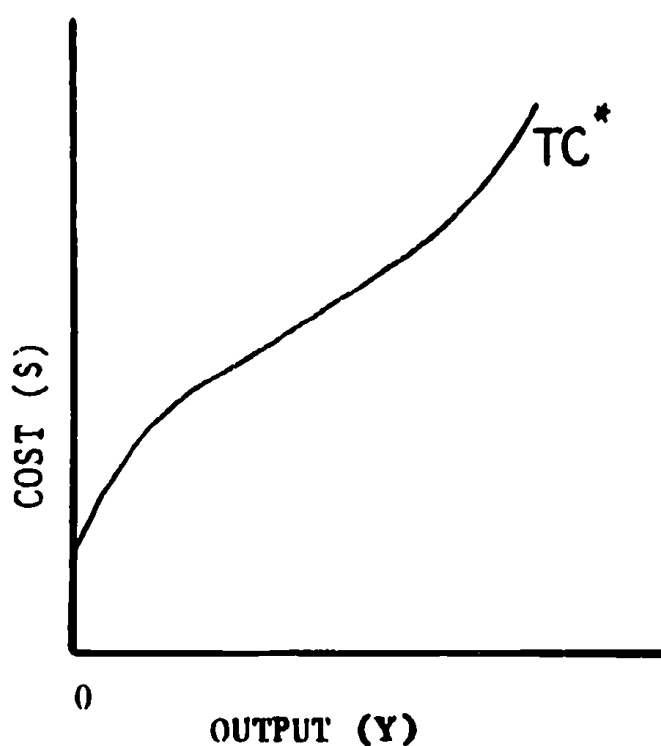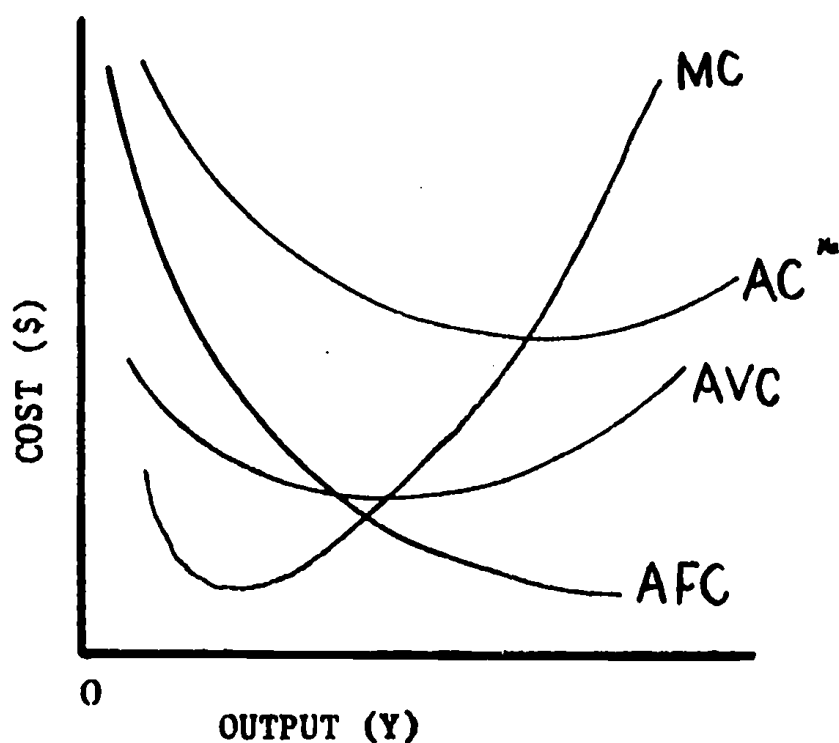
0

OUTPUT (Y)

0

OUTPUT (Y)

Figure 1

Figure 2

The above set of short run cost curves depicts the relationship between cost and output for a firm operating with one fixed input, usually assumed to be the physical plant, or facilities in general. It is possible to define and determine for each firm a specific set of short run cost curves such as those depicted above; the actual or exact shape of the curves for each firm being dependent on the particular output level and the size of the firm's facility or plant.[17] Thus, each time a firm increases or decreases the size of its facilities (scale of operations), another set of cost curves will need to be estimated or constructed if the relationship between cost and output is to be accurately measured. In the short run then, a firm's cost is a function of its output level and its scale of operations. This paper is concerned specifically with those curves in the figures above labeled TC and AC. These curves represent the firms' short run total cost and short run average or per-unit cost, respectively.

The Relationship Between Cost And Output
In The Long Run

In this section the long run average cost curve and long run total cost curve are discussed specifically. Also, the concept of economies of scale is introduced and its role in determining the shapes of the long run average and total cost curves is set out. Last, the notion of optimum scale of operations (plant) is formally presented and briefly discussed. Throughout this section two things discussed in previous sections should be remembered or kept in mind: (1) the firm can choose and construct any desired scale of plant in the long run, with all inputs becoming variable; and (2) the long run can be viewed, with reference to a specific problem, as all possible short run situations that the firm can choose to operate in.

The Long Run Average Cost Curve.[18] Let us assume for purposes of analysis that it is possible for the school district to construct only 3 different sized high schools. These are represented by $SRAC_1$, $SRAC_2$ and $SRAC_3$ in Figure 3 below. Each of the curves in Figure 3 represents the short run average cost curve for a particular and different sized high school (scale of plant).
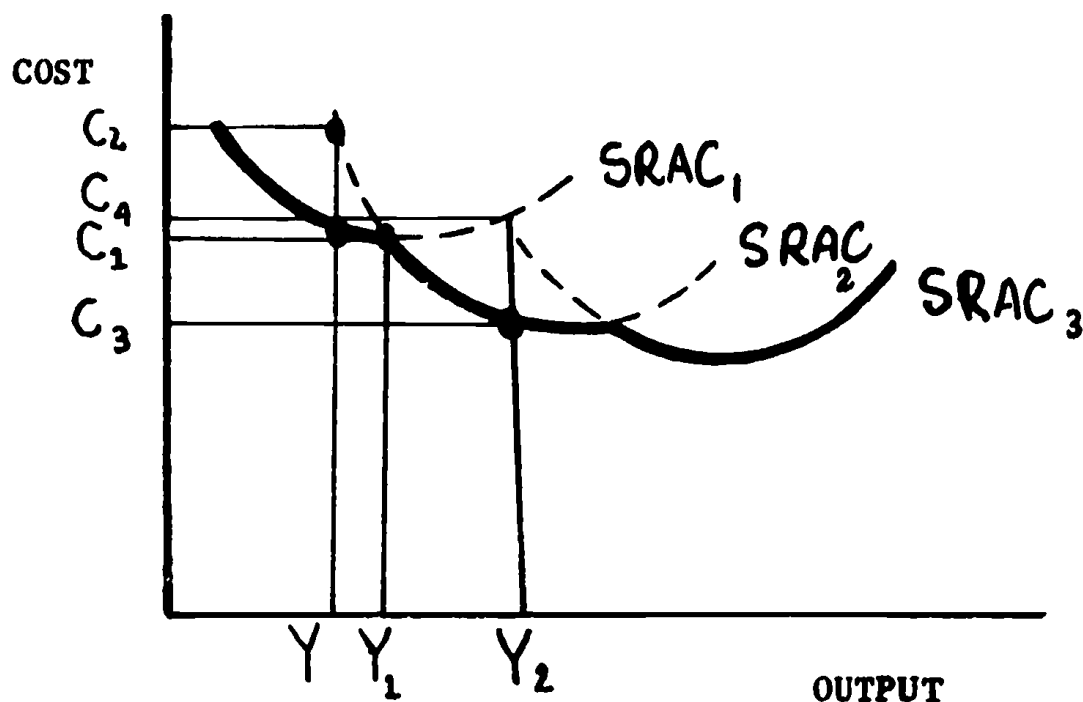

Figure 3

In the long run the school district can construct any one of the three different sized high schools. What sized high school should the district construct? The answer depends upon the long run output which is to be produced. Whatever the level of output is to be, the school district in the long run will want to produce that level of output such that per unit or per-pupil costs are minimal (for a given quality and quantity of educational services). Referring to Figure 3, assume output (or the flow of educational services) is Y or projected to be at Y level per some unit of time. The school district would construct that sized high school represented by $SRAC_1$, since it would result in a smaller per unit or per-pupil cost (YC_1) than will the other possible high school sizes. Per unit costs would be YC_2 if $SRAC_2$ were used. For output Y_1, the school district would be indifferent between $SRAC_1$ and $SRAC_2$, (unless it expected substantial increases in output in the near future; in which case, it would choose the scale of plant indicated by $SRAC_2$). For output Y_2, it would construct $SRAC_2$.

The long run average cost curve can now be defined: it gives the minimum cost per unit of producing various outputs when the firm can construct any desired scale of plant. Defined somewhat differently, the long run average cost curve "is a locus of points representing the least unit cost of producing the corresponding output.[19] In the long run the broken segments of the SRAC curves become meaningless because the firm would never operate on the broken segments since it could reduce costs by changing to a different scale of plant.

The number of different sized high schools which a school district can build in the long run are infinite. A series of SRAC curves representing seven different possible high school sizes are depicted in Figure 4: Outer

portions of the SRAC curves form a solid line which is the long run average
cost curve. Since the long run average cost curve is made up of very small
segments of the various SRAC curves, it can be considered tangent to all
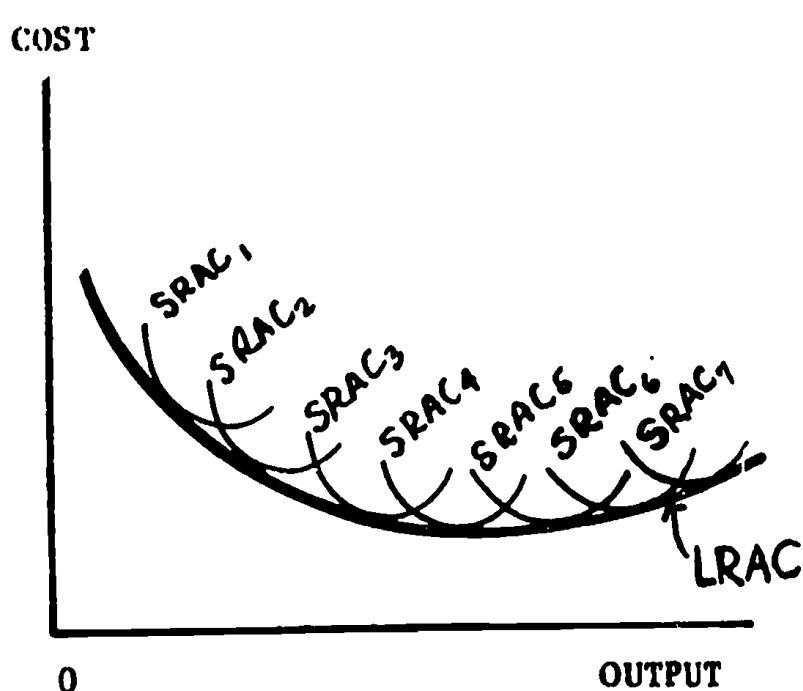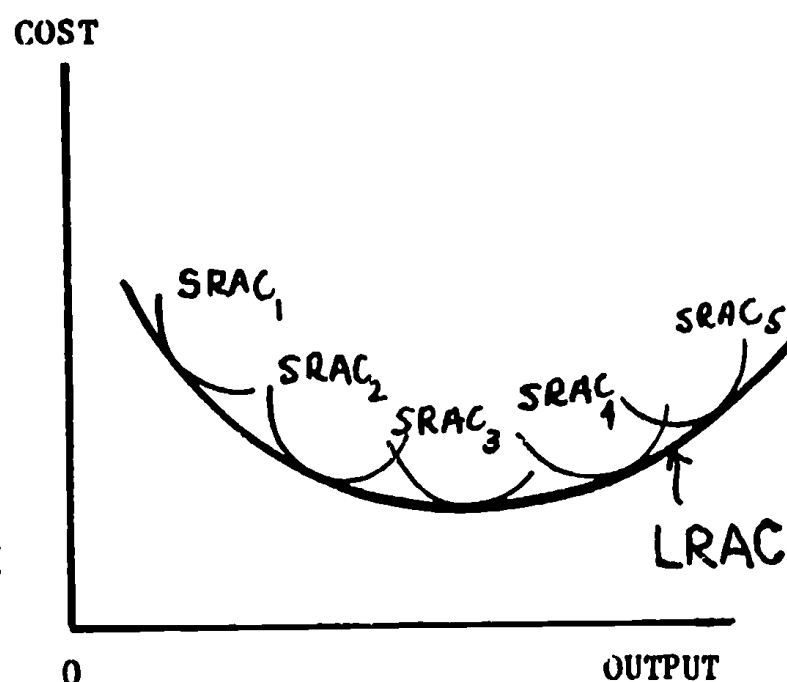

Figure 4   Figure 5

possible SRAC curves representing the different high school sizes which the
school district could conceivably construct. Mathematically, it is referred
to as the envelope curve to the SRAC curves.

The Shape of the Long Run Average Cost Curve and Economies of Scale. The
long run average cost curve in Figure 4 (LRAC) resembles a shallow U shape.
This conforms to traditional economic theory which hypothesizes a U-shaped
long run average cost curve. However, its exact shape is an empirical
question, with the factors governing its shape referred to as economies and
diseconomies of scale.

Economies of scale. The traditional U-shaped LRAC curves depicted in
Figures 4 and 5 above represent the situation where scales of plant become
more efficient up to some particular output level, and then beyond that
point become less efficient as the level of output becomes increasingly

larger. Increasing efficiency, associated with higher output levels and larger scales of plant, is reflected by SRAC curves lying at successively lower levels and farther to the right. This situation is depicted by $SRAC_1$, $SRAC_2$ and $SRAC_3$ in Figure 5. Decreasing efficiency, associated with even larger scales of plant, is depicted by $SRAC_4$ and $SRAC_5$ which lie at successively higher levels and farther to the right. Thus, the U-shaped long run average cost curve obtains.[20]

The forces causing the LRAC curve to decrease for larger outputs and scales of plant are called economies of scale. Economies of scale can be defined as net reductions in per unit costs resulting from long run expansions in a firm's output.[21] These net reductions in costs result from: (1) increased specialization and division of labor; (2) greater proficiency with increased concentration of effort; (3) reduction in excess capacity of certain inputs; (4) many inputs becoming cheaper when purchased on a larger scale; and (5) as the scale of operation expands there is usually a qualitative change in equipment.

<u>Diseconomies of Scale</u>. The question arises why, once the scale of plant is large enough to take advantage of all economies of scale, continued increases in output and scale eventually results in lesser efficiency (the U-shaped LRAC curve). Theory suggests that limitations to the efficiency of management in controlling and coordinating a single plant or firm eventually occur.[22] These limitations are called diseconomies of scale and result from: (1) an overall breakdown in the communications network; (2) the fact that administrators no longer receive the kind of information necessary for optimum decision-making; (3) administrators must begin delegating authority and responsibility to lower level and less competent individuals; and (4) the organization becomes 'topheavy'.

Thus economies and diseconomies of scale determine the shape of the long run average cost function. Therefore, in examining scale effects in the operation of public high schools in West Virginia we are ultimately concerned with the shape of the LRAC curve.

The LRAC may assume a variety of shapes depending upon the nature of economies and diseconomies of scale in a particular 'industry'. Three examples are given in Figures 6, 7, and 8:
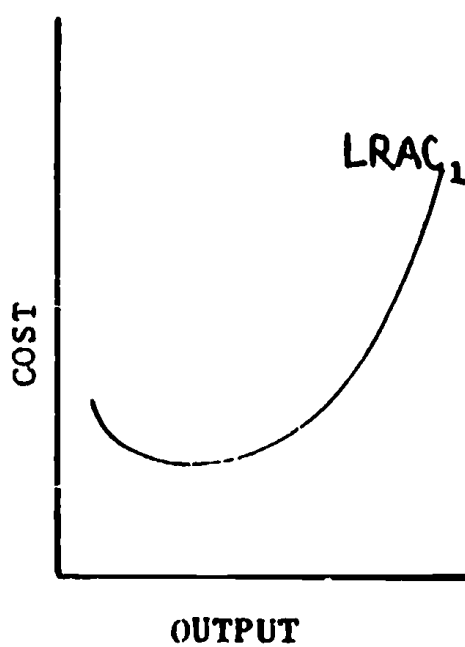
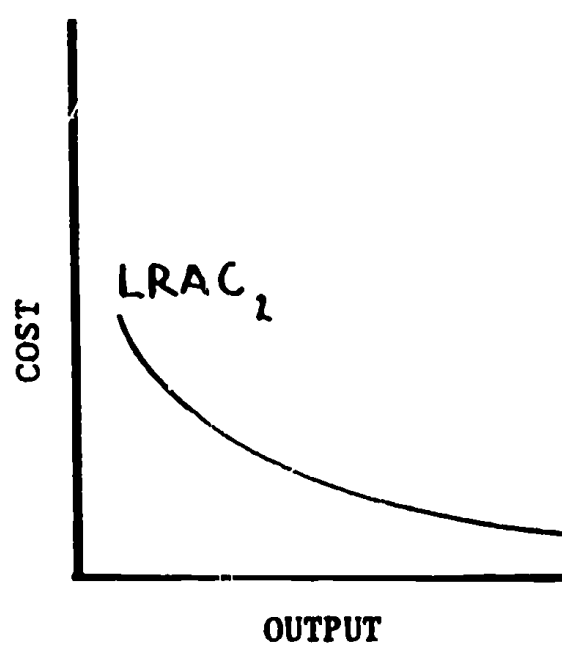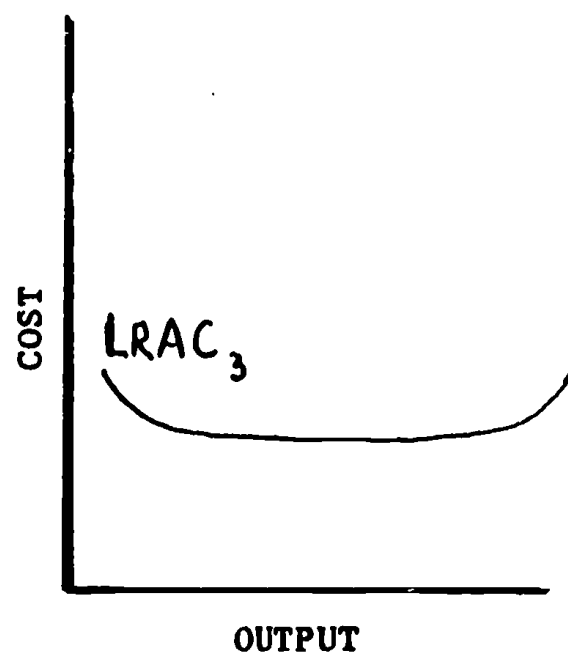

Figure 6	Figure 7	Figure 8

The optimum scale of plant. The 'optimum scale of plant' generally refers to the most efficient scale of plant that the firm can construct (where efficient refers to cost minimizing). Graphically and geometrically it represents that scale of plant with a short run average cost curve tangent to the long run average cost curve at the minimum points of each. The short run average cost curve of the optimum scale of plant is $SRAC_2$ in Figure 9, below.
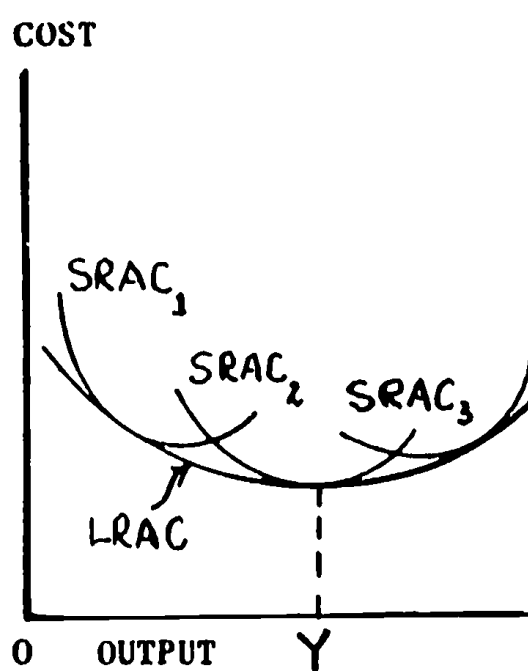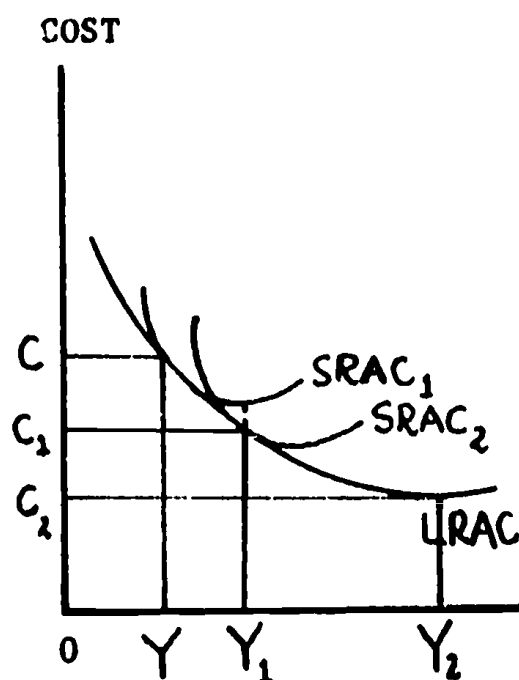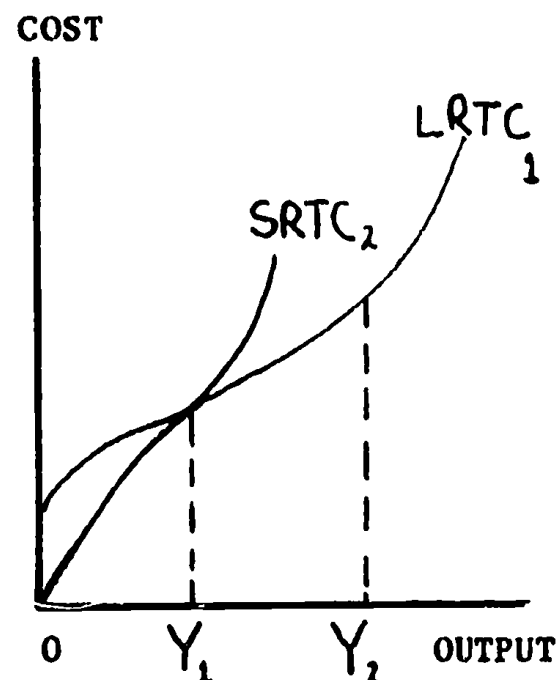
Figure 9$_a$ Figure 10   Figure 11

It should be noted that firms do not always construct the optimum scale of plant, nor do they function at the optimum output level (except under perfect competition). For example, in Figure 9$_a$ the scale of plant represented by SRAC$_2$ will produce output Y at a lower per unit cost than will any other scale of plant, and output Y can be produced at a lower cost per unit than can any other output. But for output levels greater or less than Y, per unit costs will be higher. Scales of plant other than the optimum scale of plant will produce these output levels at lower costs per unit than will the optimum scale of plant. To prove this point, consider Figure 10. Suppose the firm is producing output Y with the scale of plant SRAC$_1$. The scale of plant SRAC$_1$ is being operated at less than its optimum rate of output. Assume a change in output to Y$_1$ is contemplated. The increase in output can be achieved in two ways: (1) by increasing the output with existing scale of plant SRAC$_1$, or (2) by increasing the scale of plant and moving to SRAC$_2$.

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If (1) is chosen, $SRAC_1$ will be used at its optimum rate of output and per unit costs will be lower than C. However, if the firm chooses (2), economies of scale from the larger scale of plant will yield greater per unit cost reductions for output Y_1 than will (1). Per unit costs will be C_1, with scale of plant $SRAC_2$, and this is the lowest cost at which that output can be produced. For output levels between zero and Y_2 the firm will achieve the lowest per unit costs for any given level of output by using a less than optimum scale of plant at less than the optimum rate of output. Likewise, for any given level of output greater than Y_2, the lowest possible cost per unit will be achieved if the firm uses a greater than optimum scale of plant and produces a greater than optimum level of output. Thus to minimize cost for any given level of output, the firm should use the scale of plant whose short run average cost curve is tangent to the long run average cost curve at that particular level of output.

The Long Run Total Cost Curve

The LRTC curve of a firm can be constructed from its LRAC curve, and vice versa. Consider again Figure 10. At output levels Y, Y_1 and Y_2, long run total costs will be Y(C), $Y_1(C_1)$ and $Y_2(C_2)$ respectively. Long run total costs can be computed from other output levels in the same way. The resulting LRTC curve would look like $LRTC_1$ in Figure 11. The LRTC curve in Figure 11 suggests decreasing long run average costs between zero and Y_2 and increasing long run average costs for outputs greater than Y_2.

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Statistical And Methodological
Considerations: Educational 'Output'.
The Data, And Estimation Procedure

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The theory presented in part two of this paper suggests that in order

to determine the relationship between cost and output, and the extent to which economies of scale exist in the operation of public high schools in West Virginia, either a long run average cost curve or long run total cost curve must be estimated. Before this is attempted, however, certain other things should be discussed; specifically, the problem of 'educational output', the data employed in the study and the method used to estimate the long run cost function for selected public high schools in West Virginia.

Educational Output

For years, public school systems, researchers, academicians, interested parents and government officials have been concerned with 'educational output'. Specifically, what is educational output, how can it be defined and measured? Thus far no one has developed a universally accepted, additive and quantifiable measure which we can title 'educational output'.

Educators have set certain normative objectives such as the development of individual potential, self expression, the fulfillment of individual capacities and preparation of individuals for a democratic society.[24] These represent stated objectives. The difficulty of measuring, for a given school, the extent to which any one of these goals is realized is immediately evident.

The above represent only the stated objectives or manifest functions of education. What about the measurement of the various latent functions[25] which are supposedly performed by education? How can both the manifest and latent functions be quantified, weighted and aggregated to obtain a measure of 'educational output'?

The answer to this last question presupposes that we know what the functions of public education are precisely. Some individuals such as Hills

suggest:[26]

> ...we do not know what the functions of education are.
> Although there are volumes upon volumes of ideological exhorta-
> tions and prescriptions concerning what the functions of education
> should be, there is relatively little in the way of concrete
> knowledge concerning the actual, objective consequences of exist-
> ing educational activity. That is to say, we have a great deal
> of information regarding the subjective dispositions -- aims,
> motives, and purposes -- attributed to education, but we know
> little enough about what schools actually do, and practically
> nothing about the objective consequence of these activities for
> the larger structures in which the schools are involved.

Given the present state of knowledge regarding educational outputs and
the multiple output problem, several 'proxy' variables were considered and
one eventually chosen to represent educational output -- average daily
attendance.[27] While not as glamorous as achievement test scores, it has
been used elsewhere[28] as a proxy for educational output and is much less
restrictive and biased than achievement scores. The use of achievement
scores as a measure of educational output, a priori, limits educational out-
put to cognitive types, and is consistent with the narrowly defined normative
types of objectives and functions to which Hills refers. However, the
cognitive outputs may be only a very small part of the total educational
output.

In making average daily attendance a proxy for educational output, no
such restrictions on the nature of educational output apply; we are, instead,
stating simply, as average daily attendance increases so does the sum total
of educational experiences in the educational system. More precisely,
average daily attendance varies directly with educational output. Thus,
average daily attendance has at least two advantages within the context of
the above discussion: (1) as a measure of educational output it allows
educational output to include learning experiences beyond the cognitive

types; and (2) average daily attendance reflects indirectly both the manifest and latent functions of the public school system more nearly than other empirical measures thus far conceived.

Using average daily attendance as a proxy for educational output also has a statistical advantage. Not only is average daily attendance relatively stable and predictable over the academic year; but it is also highly related to the total cost figure. Both these characteristics reduce potential bias in the estimation of the long run cost function.

A fourth consideration suggesting use of average daily attendance is that in certain instances school boards, superintendents and other decision makers use either average daily attendance or net enrollment (which is highly correlated with average daily attendance) as a figure in various types of decisions. The most obvious examples are the state aid formulas and federal programs which base monetary allocations on either average daily attendance or net enrollment. The underlying assumption is that some minimum level of learning is occurring. Thus, in certain instances, average daily attendance is treated as output or used as a basis for various administrative decisions.

The above considerations suggest several advantages in the use of average daily attendance as a proxy for educational output. In light of existing knowledge regarding the nature and functions of our educational system, average daily attendance appears to be not only a reasonably 'good' proxy for educational output but also an appropriate one.

The Data

There are fifty-five school districts in West Virginia. Forty-five districts contain one or more senior high schools either grades nine to

twelve or ten to twelve. Data on cost and output were collected for the 1969-1970 fiscal year.

Senior high schools were chosen as the unit of analysis for essentially two reasons: (1) school districts in West Virginia and other states are (if they are not already there) moving away from the grade seven through twelve concept; thus to estimate a long run cost function and investigate the extent of economies of scale in senior high school operation is logically most relevant and has potentially greatest benefit in terms of new information for decision-makers; and (2) senior high school output or ADA is flexible and can be considered a discretionary policy variable, in that administrators at the district level have control over the various ADA levels of senior high schools (i.e., through busing, consolidation and/or investment in new facilities).

While there have been studies of scale effects at the district level, these provide only marginally useful information to school administrators, boards of education and state departments of education; because in many states the school district is synonymous with the county, as is the case in West Virginia. School administrators have no discretionary control over district size. Changes in district or county size occur through exogenous changes (i.e., industrial development or out migration). Thus, to say the optimum sized school district is thirty-five thousand students provides little relevant information to the board of education and district superintendent simply because they have no internal control or mechanism to effect district size.

Junior high schools and high schools grades seven to twelve were excluded from the study because of possible differences in organizational structure which conceivably might confound estimation of scale effects and

make interpretation of results more difficult. Thus, the study involves senior high schools grades nine through twelve and grades ten through twelve.

Data was gathered on high schools which ranged in average daily attendance from 177 students to 2130 students. Average daily attendance of 177 students represents the smallest output level while the average daily attendance of 2130 students represents the largest output level actually found in the state. Thus, the maximum possible range of output as measured by ADA was obtained.

A total short run operating cost was obtained for each high school in the sample for the 1969-1970 fiscal year. The figure represents an operating expenditure on administrative inputs, teacher inputs, instructional inputs, operation and maintenance inputs. It accounts for no less than ninety-seven percent of a school's current operating costs. The remaining three percent include expenditures on such things as auxiliary services (health programs, school lunches and other items). The figure is also net of transportation costs which were impossible to calculate accurately on a per school basis, bonded indebtedness and depreciation costs.

Qualitative differences and economies of scale. Interpretation of studies examining economies of scale is extremely difficult if wide variations in school quality exist. For instance if per pupil operating expenditures in school A are lower than in school B, but school B is qualitatively superior to school A, little meaning can be attached to a discussion of scale effects. Therefore, an attempt has been made here to minimize qualitative differences in schools to the greatest extent possible. The usual procedure is to construct a school or district 'quality index' composed of various school inputs and thereby 'control' for school quality either explicitly or implicitly by introducing this index into the cost equation

or analysis. This procedure has been employed by several individuals in examining public school expenditures and scale effects in public education.[29]

However, this procedure assumes, a priori, that the different inputs used in the various indices relate to and measure school quality. Typical input variables assumed to reflect school quality include: (1) the student-teacher ratio, (2) per pupil expenditures, (3) number of books in school library, (4) number of high school credit units offered, (5) percent of teachers with master's degrees, (6) average teacher salary, (7) percent of teachers in two or more fields.

Raymond[30] in examining the relationship between selected input variables and school quality concluded:

> Empirical results pertaining to the state of West Virginia give no support to the use of certain input variables as proxies for the quality of education. There was no evidence of a significant relationship between quality and student-teacher ratio, the percent of teachers in two or more fields, current expenditures, or the adequacy of library facilities. Thus, in spite of obvious and perhaps convincing arguments in support of these factors, it appears that, in fact, they are not always accurate indicators of quality.[31]

Raymond did find, however, that salary variables were related to school quality but warned at the same time against indiscriminant use of salaries as a proxy for school quality. Referring to the above conclusions he says of salaries:

> No such unequivocal statement relating to teaching salaries may be made. The significance of the salary variables provides some justification for the use of these variables as proxies for quality. On the other hand, it should be noted that the highest coefficient of determination between a salary variable and a quality variable was about .36. This, in conjunction with the strong relationships between salaries and population characteristics would seem to warrant only a very guarded use of salaries to measure quality....The exact nature of the relationship between salaries and quality could not be determined with available data.[32]

Making limited use of these findings, all schools whose mean teacher salary was $6250 or less were excluded from the population. It was felt this procedure would reduce large variation in quality between schools. Certain senior high schools had mean teacher salaries in the neighborhood of $5500.

With the intent of further minimizing qualitative differences among high schools studied; a second procedure, used by Riew[33] to minimize qualitative differences among schools in a stu ' of economies of scale in selected Wisconsin high schools, was also employed. Only those high schools accredited by the North Central Association of Colleges and Secondary Schools as of 1969-1970 were included in the population.

Establishing a minimum mean teacher salary and including only those senior high schools which were North Centrally accredited reduced the population size by thirteen observations. While all qualitative differences are not eliminated by these two procedures, it is felt that enough variance in school quality has been eliminated to make discussion of economies of scale meaningful for the remaining senior high schools in the study.

From a population of sixty-five senior high schools in West Virginia a purposive sample of thirty-eight observations was obtained. The sample was purposive in two ways: (1) data was obtained so as to maximize the range of observations on educational output in order to meet certain 'ideal' data criteria for cost studies; and (2) because West Virginia has relatively few schools whose average daily attendance exceeds twelve hundred students, an attempt was made to obtain all observations of schools where ADA exceeded one thousand students. The cost function estimated from this sample can be considered representative of the population and descriptive of high schools throughout the state in general.

We turn now to a discussion of the method used in estimating the long run cost function for public high schools in West Virginia. In so doing we answer the question why short run data was obtained to estimate the long run cost function.

The Method Used In Estimating
The Long Run Cost Function

The method selected to estimate the long run cost function for public high schools in West Virginia is suggested in Eads[34] and Eads, Nerlove and Raduchel.[35] The procedure minimizes certain types of bias usually associated with estimation of long run cost functions and is particularly consistent with certain characteristics indigenous to public education; thus, it provides an excellent procedure to estimate the long run cost function here. The procedure makes use of the relationship between the long run cost function and family of short run cost curves.

As indicated in part 2, the long run cost function gives the minimum cost of producing each level of output under the assumption that the firm is free to vary the size of its plant. However, as Eads, Nerlove and Raduchel indicate, "except by chance, one never observes a firm on the long run total cost function, but instead observes it on a short run total cost function. The firm is unable in the short run to adjust all of its factors of production to the optimum level for the output level it is given to produce."[36]

As Henderson and Quandt[37] suggest:

> Long run total cost is a function of output level, given
> the condition that each output level is produced in a plant
> of optimum size. The long run cost curve is not something
> apart from the short run cost curves. It is constructed from
> points on the short run cost curves. Since size is assumed

continuously variable, the long run cost curve has only one point
in common with each of the infinite number of short run cost
curves.

Thus the long run cost curve is the envelope curve of the
short run cost curves, touching each short run cost curve and
intersecting none.[38]

It is this tangential relationship and the fact that most firms are

observed on their short run cost curve that suggests the procedure for

estimation of a long run cost function. Consider the production function:

$$Y = f(X_1, X_2, \ldots X_n) \qquad (3.1)$$

where Y represents output and X_1, X_2, ..., X_n represent various inputs and

X_n the fixed input (Plant or facility). The short run cost function

obtained from (3.1) under the assumption of cost minimization can be

written as:

$$C = \phi(Y, p_1 \cdots p_{n-1}, X_n) + p_n X_n \qquad (3.2)$$

where p_1....., p_n are exogenously determined input prices. The above equation

states that total cost is a function of output level, input prices and plant

size. Writing this equation representing the family of short run cost

functions in implicit form we obtain:

$$C - \phi(Y, p_1, \ldots, p_{n-1}, X_n) - p_n X_n$$

$$= G(Y, p_1 \ldots, p_n, X_n) = 0 \qquad (3.3)$$

The condition existing at each point of tangency between a short run cost

function and the long run cost function is given by:

$$\frac{\partial G}{\partial X_n} = 0 \qquad (3.4)$$

Thus, solving (3.4) and substituting the result into (3.3) we obtain the

long fun cost function:

$$C = {}_L(Y, p_1, \ldots P_{n-1}) \qquad (3.5)$$

where long run cost becomes solely a function of output and the given

price of the inputs.[39]

The above discussion suggests an indirect method of estimating the parameters of the unobservable long run cost function. One can estimate the parameters of the family of short run cost functions and then make use of this relationship between the short run cost functions and the long run cost function to obtain the parameters of the long run cost function.[40]

Consider the following example given by Henderson and Quandt[41]. Assume a short run total cost function of the following form has been estimated:

$$SRTC = .04Y^3 - 0.9Y^2 + (11 - X_n)Y + 5X_n^2 \qquad (3.6)$$

where Y represents output and X plant size or the fixed input. Now setting the partial derivative of the implicit form of (3.6) with respect to X_n equal to zero we obtain:

$$-Y + 10X_n = 0 \qquad (3.7)$$

which has the solution $X_n = .01Y$. Substituting this result into (3.6) we obtain the long run cost function:

$$SRTC = 0.4Y^3 - 0.9Y^2 + 11 - .01Y(Y) + 5(0.1Y)^2$$
$$= .04Y^3 - 0.95Y^2 + 11Y$$

Long run average cost is obtained from dividing LRTC by output (Y):

$$LRAC = .04Y^2 - .095Y + 11.$$

The above model requires only that firms in the short run minimize costs; that is, be somewhere on its short run average cost curve for a given level of output and scale of plant.

This assumption appears to be extremely valid for public high schools in West Virginia, because in most high schools, average daily attendance has either remained relatively constant or even declined slightly since

1967. This relatively 'steady state' has allowed schools to make the necessary adjustments which place them either on their short run cost curve or the long run cost curve. Therefore, the procedure is utilized here not only because it minimizes certain types of statistical bias but also because the assumption of cost minimization appears consistent with public high school operation in West Virginia.

To estimate the total cost function the procedure of ordinary least squares was employed. It was chosen because it results in parameter estimates which have certain desired properties. More precisely, least squares estimators are best and unbiased. Further, least squares is most appropriate where: (1) deviations between planned output and actual output are small, (2) cross section data is employed and (3) the firm is only concerned with cost minimization for given levels of output.[42] All three conditions set out above apply here and thus the method of ordinary least squares was used.

Analysis, Findings And Conclusions

As suggested in section 2, we are most concerned with the shape of the long run average cost function. It is the shape of the LRAC curve which indicates the nature or extent of scale effects in a given 'industry'. The shape of any curve is dictated by the particular mathematical function specified. Thus, the question becomes what functional form best fits or describes the data.

This is not always an easy question to answer for as Klein[43] points out, frequently in econometric research more than one hypothesis is consistent with a given sample of data...."and non-linear cost functions may sometimes fit a given sample of data as well as a linear function."

Because the form of the function is crucial here the following method is employed to avoid possible misspecification. First, a linear function is estimated where cost (Y) is a function of output (X). Then second and third order terms in output (X_1^2 and X_1^3) are added and retained in the regression equation only if their coefficients prove to be, on application of the 't' test, significantly different from zero at the one percent level. That is, we will only entertain the possibility of a curvilinear total cost function if both the higher order terms differ significantly from zero at the .01 level. Usually a measure of goodness of fit is given by the R^2 value. However, because both linear and non-linear functions may give a good fit to the same data, the above procedure in addition to examination of R^2's is employed. Also the change in the mean square will be examined upon introduction of the higher order terms.

Statistical Results

Total cost functions of the following general form were estimated:

$$Y = \hat{b}_1 X_1 + \hat{b}_o + u \qquad (4.1)$$

$$Y = \hat{b}_3 X_1^3 + \hat{b}_2 X_1^2 + \hat{b}_1 X_1 + \hat{b}_o + u \qquad (4.2)$$

where X_1 equals average daily attendance and Y equals total operating cost. All total cost functions were forced through the origin.[44] The results are given in Table I below. When the additional higher order terms are introduced into the equation, suggesting a curvilinear relationship between cost and output, not only is the mean square reduced by a significant extent but both regression coefficients of the higher order terms are significantly different from zero at the .01 level. The modest increase in the R^2 means little here because we are concerned with the shape of the total cost function, in particular its departure from

TABLE I

RESULTS AND COMPARISON OF

EQUATION (4.1) AND (4.2)

Equation Number	Y	$\hat{b}_1 X_1$	$\hat{b}_2 X^2$	$\hat{b}_3 X^3$	R^2	Increase in R^2	F Value - Total	Mean Square	Mean Square Reduction
(4.1)	Y =	509.47 (10.28)*			.9867	.9867	2453	29414	
(4.2)	Y =	703.73X_1 (51.53)*	$-.37X^2$ (.08)*	$.00015X_1^3$ (.00003)*	.9928	.0060	1418	17065	11349

Y = Total operating cost
X = Average daily attendance
* Indicates regression coefficient is significant at the .01 level.
Figures in parentheses are the standard errors of the regression coefficients.

linearity, which is suggested by the significant regression coefficients of the higher order terms. It does appear, then, that the total cost function is curvilinear and equation (4.2) is chosen here.

Consistent with the methodology previously set out, we now introduce scale explicitly into the short run cost function estimated above. The variable chosen to represent plant size was total square feet of high school facility.[45] Five short run total cost functions, each specifying a different scale, output, and cost relationship, was estimated. The rationale for this rather shot gun approach lies in the complexity of second and third order models which yield 'surfaces'. As Draper and Smith[46] indicate with regards to second and third order models, "ommission of terms implies possession of definite knowledge that certain types of surface (those which cannot be represented without the omitted terms) cannot possibly occur. Knowledge of this sort is not often available. When it is, it would usually enable a more theoretically based study to be made." Of the five equations specified and estimated, the following general form was selected:

$$Y = \hat{b}_3 X_1^3 + \hat{b}_2 X_1^2 + \hat{b} X_1 + \hat{c}_1 X_1 X_2 + \hat{c}_2 X_2^2 + u \qquad (4.3)$$

where Y equals total operating cost, X_1 equals average daily attendance and X_2 equals total square feet of high school facility.

Estimation of the above generalized form resulted in the following specific short run total cost function for selected public high schools in West Virginia:

$$SRTC = 696.37 X_1 - .44672 X_1^2 + .00013 X_1^3 \qquad (4.4)$$

$$(59.52)^* \qquad (.127)^* \qquad (.00004)^*$$

$$+ .00174 X_1 X_2 - .00001 X_2^2$$

$$(.00134) \qquad (.00001) \qquad R^2 = .9934$$

Again, the regression coefficients of average daily attendance are significant

at the .01 level. The total F of the equation is 872.08. Also, inclusion

of the scale terms reduced the mean square from 17065 to 16641.

Following the procedures set out in section 3, the long run total cost

function for selected public high schools in West Virginia was obtained:

$$LRTC_a = 696.37X_1 - .37103X_1^2 + .00013X^3 \qquad (4.5)$$

Dividing the above function through by X_1 gives the long run average cost

function:

$$LRAC_a = 696.37 - .37103X_1 + .00013X^2 \qquad (4.6)$$

Given the empirically estimated long run average cost function for

selected high schools in West Virginia, is there some optimum scale of plant?

In section 2 the optimum scale of plant was given to be the most efficient

sized facility that could be constructed assuming variability of all inputs:

efficiency being synonymous with minimum cost for a given level of educational

quality. Graphically and geometrically it was said to be the scale of plant

resulting in a short run average cost curve whose minimum point also forms

the minimum point on the long run average cost curve. The question thus

becomes does the LRAC function estimated above have a minimum point or value?

The estimated long run average cost function for public high schools in

West Virginia was found to satisfy both the necessary and sufficient conditions

for a minimum value. Specifically, the minimum point on the long run average

cost function occurs where educational output, as measured by average daily

attendance, is 1426 pupils. That is, for a particular high school size when

average daily attendance is 1426 pupils, the cost per pupil in average daily

attendance will be minimum, or the lowest possible value.

To determine the specific scale which gives the tangency of both the

short run and long run cost curves at their minimum points we must
return to the estimated short run cost function. Performing certain opera-
tions the optimum scale of plant for the optimum output level of 1426 pupils
is found to be 124,062 square feet. At this point the short run and long
run and cost curves are tangent at the minimum point of both and long run
average cost or cost per pupil in average daily attendance will be minimum.

Having determined the optimum sized high school and optimum output level
let us now examine the actual shape of the LRAC curve over a relevant range
of educational output. The long run average cost curve depicted in Figure 12
represents the estimated $LRAC_a$ function in (4.6). The curve has been drawn
for educational output ranging from one hundred students in average daily
attendance to three thousand students in average daily attendance. As indi-
cated previously, actual educational output for the population of sixty-five
schools ranged from 177 students in average daily attendance to 2,131 in
average daily attendance.

The shape of the long run average cost curve supports the maintained
hypothesis in economic theory of a U-shaped long run average cost curve.
It indicates that high schools will experience net economies of scale as they
increase their output and scale of operations from an average daily attendance
of 100 pupils up to 1426 pupils. Beyond this level of output, diseconomies
of scale outweigh economies of scale and long run average cost begins to rise.

Table II below gives an indication of the effect of scale on high
school operating cost. Specifically, it shows the change in long run average
cost resulting from equal incremental changes in output. We see that sub-
stantial economies of scale can be realized from increasing educational output
at the lower levels of output. Table II indicates that a high school which
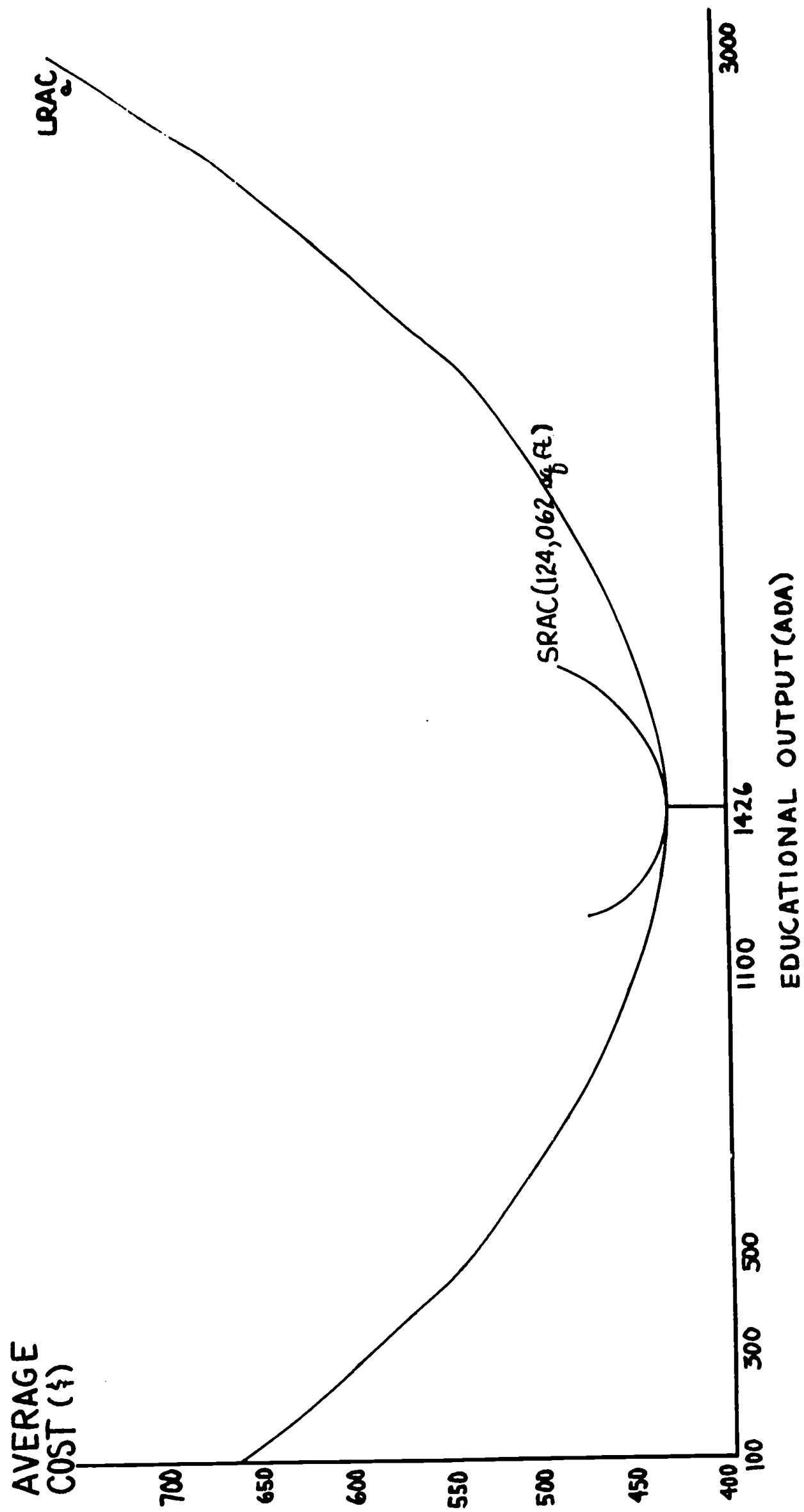increases its average daily attendance from one hundred students to three

FIGURE 12

TABLE II

THE RELATIONSHIP BETWEEN SCALE AND
LONG RUN AVERAGE COST

ADA	LRAC	Δ LRAC
100	$661	$
300	597	-64
500	543	-54
700	500	-43
900	468	-32
1100	446	-22
1300	434	-12
1500	432	- 2
1700	441	+ 9
1900	461	+20
2100	491	+30
2300	531	+40
2600	610	+79
2900	713	103

Where: (1) LRAC equals total cost
divided by pupils in average
daily attendance
(2) Δ means change in

hundred students can reduce its average cost per pupil by $64. Similarly, substantial economies of scale are realized when the high school increases its average daily attendance from 300 to 500 students. However, as output and scale continue to increase reductions in long run average cost become proportionately lesser. This suggests that economies of scale are becoming exhausted and/or diseconomies of scale more prevalent. Finally, diseconomies of scale become significant enough to outweigh economies of scale as output and plant size continue to increase. Beyond this point, increases in output and scale result in increased average cost.

It is interesting to note that virtually all high schools in the sample could reduce per pupil operating costs by changing their scale of operations and ADA levels; with over ninety percent of the sample being on the downward sloping portion of the LRAC curve, the remainder being on the upward sloping portion of the curve. Last, the model is capable of generating for any given level of output, the optimum sized facility which would place the school on the long run cost function.

FOOTNOTES

1. J. Alan Thomas, The Productive School: A Systems Analysis Approach to Educational Administration (New York: John Wiley & Sons, 1971), p. 31.

2. A number of factors support this observation: (1) the increasing percentage of school bond referendums which fail to obtain the necessary support; (2) district wide teacher layoffs which either go unquestioned by the public or are actually supported by them; (3) the reduction or elimination of extra-curricular programs, most noteably athletic and band programs; and (4) the current popularity of certain books which are highly critical of education and its practices, processes, etc.

3. An 'enlightened efficiency' factor is any organizational variable, process or relationship which can be manipulated or changed (directly or indirectly) so as to produce a net reduction in operating cost, which leaves organizational output qualitatively and quantitatively either unchanged or improved.

4. A firm is defined here as a technical unit or organization with institutional characteristics where commodities are produced. This definition allows us to begin thinking of the high school as an 'educational firm' which utilizes a variety of productive inputs per academic year to produce a certain flow of educational output.

5. C. E. Ferguson, Microeconomic Theory, (Homewood, Illinois: Richard D. Irvin, 1966), pp. 107-108.

6. James A. Henderson and Richard E. Quandt, Microeconomic Theory: A Mathematical Approach, (New York: McGraw Hill, 1958), pp. 42-43.

7. Ferguson, op. cit., pp. 107-108.

8. Jacob Viner, "Cost Curves and Supply Curves", A. E. A. Readings In Price Theory, ed. Stigler and Boulding (Homewood, Illinois: Richard D. Irvin, Inc., 1952), p. 202.

9. George J. Stigler, The Theory of Price (New York: Macmillan Company, 1966), pp. 134-135.

10. Ferguson, op. cit., p. 108.

11. Viner, op. cit., p. 205.

12. Richard H. Leftwich, The Price System And Resource Allocation, 3rd ed., (New York: Holt, Rinehart and Winston, 1965), p. 130.

13. Ferguson, op. cit., p. 108.

14. Ibid., p. 176.

15. These cost curves depict a simplified situation, often assumed for purposes of analysis, where: (1) the production process involves only two inputs, one fixed and the other variable; (2) diminishing marginal and average product eventually occur at some point for the variable

factor; and (3) constant input prices are assumed. These assumptions or conditions are consistent with the notion or definition of the short run given earlier.

16. Henderson and Quandt, op. cit., p. 56.

17. The size of the facility is usually referred to in economics as the scale of operations. The two terms will be used interchangeably throughout the remainder of the paper and can be considered synonymous.

18. Henceforth, our discussion wherever possible will be in terms of high schools rather than firms, and high school facility size rather than firms, and high school facility size rather than plant size. I consider it appropriate at this point to make a 'conceptual transformation' from firms to high schools and plant size to facility size. This will bring the discussion closer to reality and also more readily indicate to educational administrators the utility and applicability of the information obtained from the long run average cost function.

19. Ferguson, op. cit., p. 179.

20. Richard H. Leftwich, op. cit., pp. 143-144.

21. This definition of economies of scale is based on and essentially consistent with the definition given by Jacob Viner in "Cost Curves and Supply Curves," A. E. A. Readings In Price Theory, Volume VI, ed. Stigler and Boulding (Homewood, Illinois: Richard D. Irvin, Inc., 1952), p. 212.

22. Richard H. Leftwich, op. cit., p. 145.

23. Ibid., pp. 146-149.

24. John S. Brubacher, A History of The Problem of Education, (2nd ed.; New York: McGraw Hill, 1966), pp. 1-22.

25. The distinction between manifest and latent functions as a useful analytical construct comes from Robert K. Merton, Social Theory And Social Structure, (New York: The Free Press, 1968), pp. 73-138.

26. Jean Hills, "The Functions of Research For Educational Administration", Journal of Educational Administration, Volume 5, (No. 1, 1967), pp. 11-12.

27. ADA is obtained by dividing the total number of days all students have attended by the total number of school days.

28. Elchanan Cohn, "Economies of Scale In Iowa High School Operations", Journal of Human Resources, Volume III, (No. 4, 1968).

29. John Riew, "Economies of Scale in High School Operations," Review of Economics and Statistics (August, 1966), pp. 280-287; Finnis Welch, "The Valuation of Human Capital" in Papers and Proceedings of the American Economic Association, American Economic Review (May, 1966), pp. 379-392; Werner Z. Hirsch, "Determinants of Public Education Expenditures," National Tax Journal (March, 1960), pp. 29-40.

30. Richard Raymond, "Determinants of the Quality of Primary and Secondary Public Education in West Virginia," Journal of Human Resources,Vol. 3, No. 4 (1968), pp. 450-470.

31. Richard Raymond, op. cit., p. 469.

32. Raymond, Ibid.

33. John Riew, op. cit., p. 281.

34. George C. Eads, "A Cost Function for the Local Service Airlines," (Unpublished dissertation, Department of Economics, Yale University, 1968).

35. George Eads, Marc Nerlove, and William Raduchel, "A Long Run Cost Function For the Local Service Airline Industry: An Experiment in Non-Linear Estimation," Review of Economics and Statistics, Volume 51, No. 3 (August, 1969).

36. Eads, Nerlove and Raduchel, op. cit., p. 259.

37. Henderson and Quandt., op. cit., p. 60.

38. Ibid.

39. The above comes directly from Eads, et. al., op. cit., p. 260 and parallels Henderson and Quandt, op. cit., p. 60.

40. Ibid.

41. Henderson and Quandt., op. cit., p. 61.

42. A detailed discussion of the use of least squares procedures in examining and estimating cost-output relationships is given in J. Johnston, op. cit., pp. 30-43.

43. Lawrence Klein, An Introduction To Econometrics, (Englewood Cliffs: Prentice Hall, 1962), p. 121.

44. Initially, the total cost functions were estimated without being forced through the origin. It eventually occurred to me, however, that a better fit might be obtained if equations were forced through the origin because this seemed more consistent with the cost data collected. In this instance, intuition proved correct. Forced equations resulted in a better fit; evidenced by a higher R^2 and significant increase in the total F value.

45. This figure was obtained for each high school in the study from the West Virginia Rating Bureau, Charleston, West Virginia.

46. N. R. Draper and H. Smith, Applied Regression Analysis, (New York: John Wiley and Sons, 1966), p. 130.